Projectile Motion Review and Problems

All projectile motion problems are the same in theory, but in order to make it simple there are 3 types of projectile motion problems. Below are the types, a quick discussion of each, and example problems.

Remember for **ALL** projectile motion problems:

- X – direction:
  - $a_x = 0\ m/s^2$ because there are no forces acting horizontally in the x-direction.
  - Since there is no acceleration in the x-direction $v_{ix} = v_{fx}$
  - Since there is no acceleration in the x-direction: $\Delta x = v_{ix}t$
- Y – direction:
  - $a_y = -9.8\ m/s^2$ because the force of gravity is acting on the object constantly downward.
  - $v_{fy} = v_{iy} + a_y t$
  - $v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$

4 types of projectile motion

1. Projectile fired ***horizontally***
2. Projectile fired at an ***angle*** and landing at the **same elevation** compared to where it was fired. ($\Delta y = 0$ m)
3. Projectile fired at an ***angle*** and landing at an elevation ***above*** where the object was fired (**Δy is positive**)
4. Projectile fired at an ***angle*** and landing at an elevation ***below*** where the object was fired (**Δy is negative**)

1. Projectile fired ***horizontally***
   a. For an object fired horizontally, all of the initial velocity is in the x-direction. That means that $v_{iy} = 0\ m/s$.
   b. Since part a. is true, $\Delta y = ½\ a_y t^2$

Problems

i. An airplane is traveling at an initial velocity of 67 m/s horizontally at an altitude of 500 m. It drops a box of supplies when it is directly over a polar bear.
   a. How long will the box be in the air? **[10.1 s]**
   b. How far from the polar bear will it land? **[677 m]**
   c. What is the resultant final velocity (magnitude and direction) of the box when it strikes the ground? **[120 m/s]**
   d. What path does the box take as seen from a person standing on the ground (draw a picture below?
   e. Where is the plane with respect to the box when it hits the ground (what does a person in the plane see as the box leaves)?

ii. Indiana Jones is running from his enemies. He is on top of a cliff that is 9 m above a horse that is awaiting below. If the horse is 1.75 m from the wall of the cliff, how fast does he need to run horizontally in order to land on the horse and escape? **[1.29 m/s]**

3 Types of Projectiles fired at an ***angle*** (find the components of the initial velocity 1st)

a. $\mathbf{v_{ix} = v_i cos\theta}$

b. $\mathbf{v_{iy} = v_i cos\theta}$

2. Projectile fired at an ***angle*** and landing at the **same elevation** compared to where it was fired. (**Δy = 0 m**)

Problems

i. A soccer player kicks a stationary ball, giving it a speed of 10 m/s at an angle of 30 degrees to the horizontal (like #2 on Study Guide Chpt 3 Sct 3).
   a. What is the maximum height reach by the ball? **[1.3 m]**
   b. What is the total time the ball is in the air? **[1.02 s]**
   c. What is the ball's range? **[8.8 m]**
   d. What is the ball's resultant final velocity as it hits the ground?

ii. An archer wishes to shoot an arrow at a target at eye level (same elevation) a distance of 50 m away. If the initial speed imparted on the arrow is 70 m/s, what angle should the arrow make with the horizontal as it is being shot? **[2.9°]**

iii. A football is kicked 60 m horizontally. If the ball is in the air 5 s, with what initial velocity was it kicked? **[27.3 m/s @ 63.9° above the horizontal]**

iv. A football player punts a football at a 50 degree angle and it travels horizontally 45 m.
   a. What is the initial velocity of the football? **[21.2 m/s]**
   b. At what other angle can the player punt and get the same range. **[40°]**

3. Projectile fired at an ***angle*** and landing at an elevation ***above*** where the object was fired (**Δy is positive)**

Problems

i. A 2.05-m tall basketball player takes a shot when he is 6.02 m from the basket (3-point line). If the launch is at an angle of 25 degrees and the ball was launched at the level of the player's head, what must be the released speed of the ball for the player to make the shot? The basket is 3.05 m tall. **[10.9 m/s]**

ii. A field goal kicker is lining up for a field goal. If he kicks the ball with an initial speed of 20 m/s at an angle of 37°. If the crossbar of the field goal post is 3 m above the ground, find:
   a. Maximum height of the football above the ground. **[7.35 m]**
   b. Total time in flight. **[2.44 s]**
   c. At what maximum range from the field goal post can the kicker successfully get the ball over the cross bar? **[34.6 m]**
   d. How far past the cross bar with the football go if it barely misses the cross bar? **[4.6 m]**

4. Projectile fired at an ***angle*** and landing at an elevation ***below*** where the object was fired **(Δy is negative)**

Problems

i. Florence Griffith-Joyner of the U.S. set the women's world record for the 200 m run by running with an average speed of 9.37 m/s. Suppose she wants to jump over a river. She runs horizontally from the rivers higher bank at 9.37 m/s and lands on the edge of the opposite bank. If the difference in height between the two banks is 2 m. How wide is the river?

ii. A stone is thrown from the top of a bridge at a 30° angle to the horizontal with an initial speed of 20 m/s into the river below. If the height of the bridge is 45 m, find:
   a. The time the stone is in flight. **[4.22 s]**
   b. The range of the stone. **[73 m]**
   c. The velocity (horizontal, vertical, and resultant) of the stone just before it hits the ground. **[v_{fx} = 17.3 m/s, v_{fy} = -31.4 m/s, v_f = 35.9 m/s @ 61 degrees below the x-axis]**

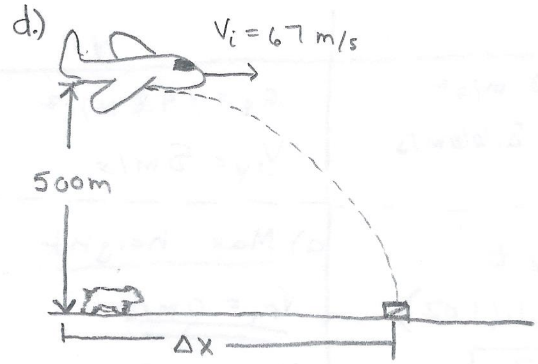
If you want extra practice refer to your book pg 98 -100 OR finish the study guide problems!

HAVE A GREAT FALL BREAK!!!

http://tlc.howstuffworks.com/family/how-to-draw-a-squirrel.htm

1. Horizontal Problems

1\.



X	Y
$a_x = 0$ m/s²	$a_y = -9.8$ m/s²
$V_{ix} = 67$ m/s	$V_{iy} = 0$ m/s
	$\Delta y = -500$ m

b.) $\Delta x = V_{ix} t$

$\Delta x = 67(10.1)$

$\boxed{\Delta x = 677 \text{ m}}$

a.) $\Delta y = \frac{1}{2} a_y t^2$

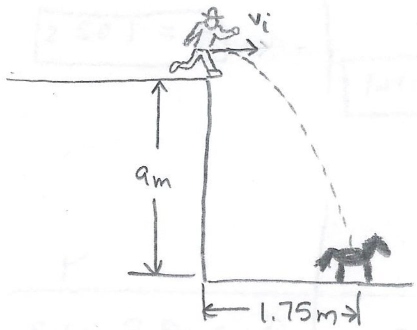
$-500 = -4.9 t^2$

$102 = t^2$

$\boxed{t = 10.1 \text{ s}}$

e.) the plane will be directly over the box. It moves with a const vel. in the x direction so in 10.1 s (when box hits the ground → $\Delta x_{plane} = 67(10.1)$ $\Delta x_{plane} = 677$ m also. A person on the plane looking down would see the box the entire time.

2\.



X	Y
$a_x = 0$ m/s²	$a_y = -9.8$ m/s²
$\Delta x = 1.75$ m	$V_{iy} = 0$ m/s
	$\Delta y = -9$ m

$\Delta x = V_{ix} t$

$1.75 = V_{ix}(1.35)$

$\boxed{V_{ix} = 1.29 \text{ m/s}}$

$\Delta y = \frac{1}{2} a_y t^2$

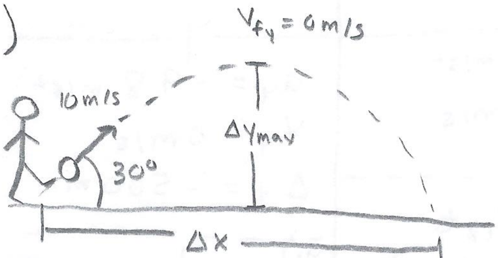
$-9 = -4.9 t^2$

$1.83 = t^2$

$t = 1.35$

2. Projectiles fired at Angle SAME ELEVATION

i)



Step 1

$V_{ix} = V_i \cos 30° = 10\cos 30°$

$V_{iy} = V_i \sin 30° = 10 \sin 30°$

X	Y
$a_x = 0\ m/s^2$	$a_y = -9.8\ m/s^2$
$V_{ix} = 8.66\,m/s$	$V_{iy} = 5\,m/s$

a.) Max height

$V_{fy} = 0\,m/s$

$V_{fy}^2 = V_{iy}^2 + 2a_y \Delta Y_{max}$

$0^2 = 5^2 + 2(-9.8)\Delta Y_{max}$

$\boxed{\Delta Y_{max} = 1.3\,m}$

b.) Total time

$\Delta y = 0\,m$

$\Delta y = V_{iy} t + \frac{1}{2} a_y t$

$0 = 5t - 4.9t^2$

$-5\cancel{t} = -4.9t^{\cancel{2}}$

$\boxed{t = 1.02\,s}$

c.) $\Delta X = V_{ix}\, t$

$\Delta X = 8.66(1.02)$

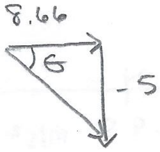
$\boxed{\Delta X = 8.8\,m}$

d.) Final Velocities

$V_{ix} = \boxed{V_{fx} = 8.66\ m/s}$

$V_{fy} = V_{iy} + a_y t \rightarrow V_{fy} = 5 + (-9.8)(1.02)$

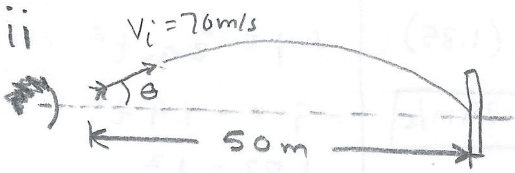
$\boxed{V_{fy} = -5\,m/s}$



$V_f = \sqrt{V_{fx}^2 + V_{fy}^2} = \boxed{10\ m/s}$

$\theta = \tan^{-1}\left(\frac{5}{8.66}\right) = \boxed{30° \text{ below horizontal}}$

ii



Step 1

$V_{ix} = V_i \cos\theta = 70\cos\theta$

$V_{iy} = V_i \sin\theta = 70\sin\theta$

X	Y
$a_x = 0\,m/s^2$	$a_y = -9.8\ m/s^2$
$V_{ix} = 70\cos\theta$	$V_{iy} = 70\sin\theta$
$\Delta X = 50$	$\Delta y = 0$

1 $\Delta y = V_{iy} t + \frac{1}{2} a_y t^2$

$0 = 70\sin\theta t - 4.9t^2$

$-70\sin\theta \cancel{t} = -4.9t^{\cancel{2}}$

$\boxed{t = \frac{70\sin\theta}{4.9}}$ (Plug into 2)

2 $\Delta X = V_{ix} t$

$50 = 70\cos\theta\, t$

$50 = 70\cos\theta\left(\frac{70\sin\theta}{4.9}\right)$

$50 = \frac{70^2 \cos\theta \sin\theta}{4.9} \times \left(\frac{2}{2}\right)$

$9.8 \cdot 50 = \frac{70^2 \cdot 2\sin\theta\cos\theta}{\cancel{9.8}}$

$2\sin\theta\cos\theta = \sin(2\theta)$

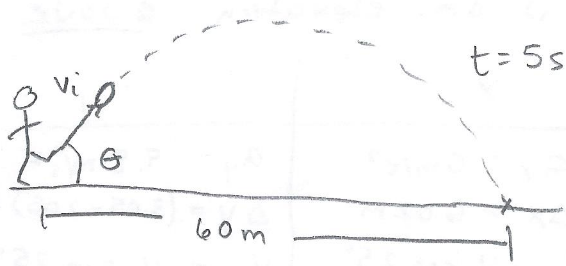
$490 = 4900\,(2\sin\theta\cos\theta)$

$0.1 = \sin(2\theta)$

$\sin^{-1}(0.1) = 2\theta$

$\boxed{\theta = 2.87°}$

iii)



$V_{ix} = V_i \cos\theta$

$V_{iy} = V_i \sin\theta$

X	Y
$a_x = 0\,m/s^2$	$a_y = -9.8\,m/s^2$
$\Delta x = 60m$	$\Delta y = 0m$
$V_{ix} = V_i \cos\theta$	$V_{iy} = V_i \sin\theta$
$t = 5$	$t = 5$

★ 2 Eq
2 Unkown
Solve for 1, plug in other.

①
$\Delta y = V_i \sin\theta t + \frac{1}{2} a_y t^2$

$0 = V_i \sin\theta t - 4.9t^2$

$0 = 5V_i \sin\theta - 4.9(5)^2$

$122.5 = 5 V_i \sin\theta$

$24.5 = V_i \sin\theta$

$\boxed{V_i = \frac{24.5}{\sin\theta}}$ put in ②

②
$\Delta x = V_{ix} t$

$\frac{60}{5} = V_i \cos\theta \frac{(5)}{5}$

$12 = \left(\frac{24.5}{\sin\theta}\right)\cos\theta$

$12 = 24.5\left(\frac{\cos\theta}{\sin\theta}\right)$

$0.5 = \frac{1}{\tan\theta}$

$\tan\theta = 2$

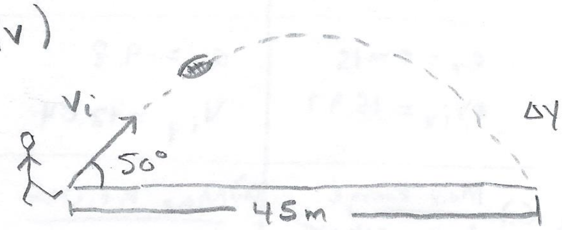
$\theta = \tan^{-1}(2)$

$\boxed{\theta = 63.4°}$

$V_i = \frac{24.5}{\sin 63.4°}$

$\boxed{V_i = 27.4\,m/s}$

iv)



$V_{ix} = V_i \cos 50°$

$V_{iy} = V_i \sin 60°$

X	Y
$a_x = 0\,m/s^2$	$a_y = -9.8\,m/s^2$
$\Delta x = 45m$	$V_{iy} = V_i \sin 50°$
$V_{ix} = V_i \cos 50°$	$\Delta y = 0m$

a)
$\Delta x = V_{ix} t$

2 Eq + 2 unknown → $45 = V_i \cos 50 t$

$45 = V_i \cos 50 \left(\frac{V_i \sin 50}{4.9}\right)$

$45 = \frac{V_i^2 \cos 50 \sin 50}{4.9}$

$220.5 = V_i^2 (0.49)$

$V_i^2 = 447.8\,m^2/s^2$

$\boxed{V_i = 21.2\,m/s}$

$\Delta y = V_{iy} t + \frac{1}{2} a_y t^2$

$0 = V_i \sin 50 t - 4.9t^2$

$4.9t^{\cancel{2}} = V_i \sin 50 \cancel{t}$

$\boxed{t = \frac{V_i \sin 50}{4.9}}$

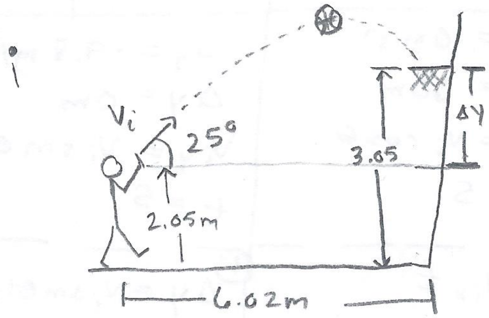
$t = \frac{21.2 \sin 50}{4.9}$

$t = 3.3s$

b.) Complimentary angles give same range. So the compliment of 50° is $\boxed{40°}$. Check $\Delta x = V_{ix} t = V_i \cos 50 t = 21.2 \cos 50 (3.3) = 45m$ ✓

3. Projectile fired at an angle at an elevation above

i



Step 1

$V_{ix} = V_i \cos 25°$

$V_{iy} = V_i \sin 25°$

X	Y
$a_x = 0\ m/s^2$	$a_y = -9.8\ m/s^2$
$\Delta x = 6.02m$	$\Delta y = (3.05 - 2.05) = {}^{+}1m$
$V_{ix} = V_i \cos 25°$	$V_{iy} = V_i \sin 25°$

1

$\Delta x = V_{ix} t$

$6.02 = V_i \cos 25 t$

$\boxed{V_i = \frac{6.02}{\cos 25 t}}$ put in 2

$V_i = \frac{6.02}{\cos 25 (0.61)}$

$\boxed{V_i = 10.9\ m/s}$

2

$\Delta y = V_{iy} t + \frac{1}{2} a_y t^2$

$1 = V_i \sin 25 t - 4.9 t^2$

$1 = \left(\frac{6.02}{\cos 25 \cdot \cancel{t}}\right) \sin 25 \cancel{t} - 4.9 t^2$

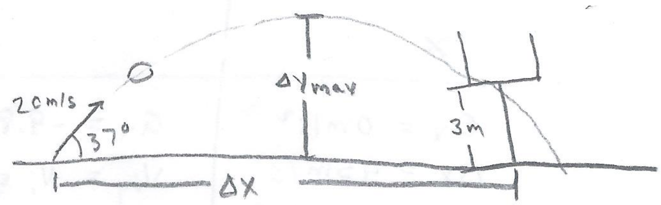
$1 = 6.02 \left(\frac{\sin 25}{\cos 25}\right) - 4.9 t^2$

$1 = 6.02 (0.46) - 4.9 t^2$

$4.9 t^2 = 2.81 - 1$

$\boxed{t = 0.61 s}$

ii



1

$V_{ix} = V_i \cos \theta = 20 \cos 37° = 15.97\ m/s$

$V_{iy} = V_i \sin \theta = 20 \sin 37° = 12.04\ m/s$

Max Range to ground
(use $t = 2.46s$)

$\Delta x_{max} = V_{ix} t$

$\Delta x = 15.97 (2.46)$

$\Delta x = 39.2\ m$

X	Y
$a_x = 0\ m/s$	$a_y = -9.8$
$V_{ix} = 15.97$	$V_{iy} = 12.04$

c.) Max Range to crossbar
(use time to crossbar $t = 2.17$)

$\Delta x = V_{ix} t$

$\Delta x = 15.97 (2.17)$

$\boxed{\Delta x = 34.7m \text{ to crossbar}}$

a.) Max height

$V_{fy} = 0$

$V_{fy}^2 = V_{iy}^2 + 2 a_y \Delta y$

$0 = 12.04^2 + 2(-9.8) \Delta y$

$\boxed{\Delta y = 7.4m}$

b.) Total time in flight (to ground)

$\Delta y = 0$

$4.9 t^{\cancel{2}} = 12.04 \cancel{t}$

$\boxed{t = 2.46 s}$

c.) Time to crossbar

$\Delta y = V_{iy} t + \frac{1}{2} a_y t^2$

$3 = 12.04t - 4.9t^2$

$4.9t^2 - 12.04t + 3 = 0$

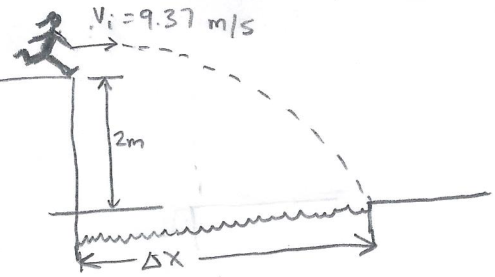
use quad.

$\boxed{t = 2.17s}$

d.) So the ball goes 34.7m to crossbar + 39.2 m total. Goes past crossbar

$39.2 - 34.7 = \boxed{4.6\ m \text{ past crossbar}}$

4: i)



☆ This is a horizontal problem! I put it in the wrong place.

X	Y
$a_x = 0 \text{ m/s}^2$	$a_y = -9.8 \text{ m/s}^2$
$V_{ix} = 9.37$ m/s	$V_{iy} = 0$ m/s
	$\Delta y = -2$ m

X:

$\Delta x = V_{ix}t + \cancel{\frac{1}{2}a_x t^2}$

$\Delta x = V_{ix}t$

$\Delta x = 9.37(0.63)$

$\boxed{\Delta x = 5.97 \text{ m}}$

Y:

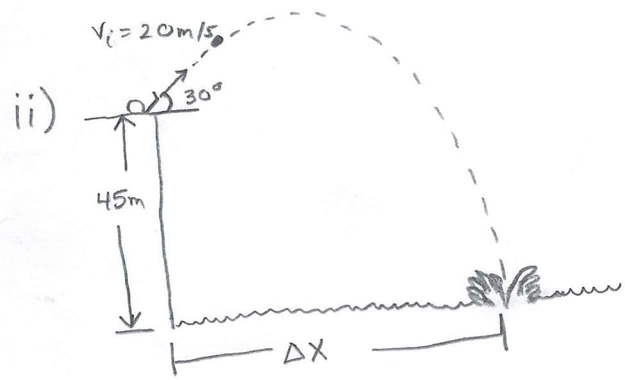
$\Delta y = \cancel{V_{iy}t} + \frac{1}{2}a_y t^2$

$-2 = \frac{1}{2}(-9.8)t^2$

$-2 = -4.9t^2$

$\boxed{0.63\text{s} = t}$

ii)



X	Y
$a_x = 0 \text{ m/s}^2$	$a_y = -9.8 \text{ m/s}^2$
$V_{ix} = 17.3$ m/s	$V_{iy} = 10$ m/s
	$\Delta y = -45$ m

b) $\Delta x = V_{ix}t + \cancel{\frac{1}{2}a_x t^2}$

$\Delta x = 17.3(4.22)$

$\boxed{\Delta x = 73 \text{ m}}$

a) $\Delta y = V_{iy}t + \frac{1}{2}a_y t^2$

$-45 = 10t - 4.9t^2$

$-4.9t^2 + 10t + 45 = 0$

☆ use quad form.

$a = -4.9$

$b = 10$

$c = 45$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-10 \pm \sqrt{100 - 4(-4.9)(45)}}{2(-4.9)}$

$t = \frac{-10 \pm \sqrt{100 + 882}}{-9.8}$

$t = \frac{-10 \pm 31.34}{-9.8}$

$t = \cancel{-2.18\text{s}}$ OR $\boxed{4.22\text{s}}$

$V_{ix} = V_i \cos\theta = 20\cos 30° = 17.3$ m/s

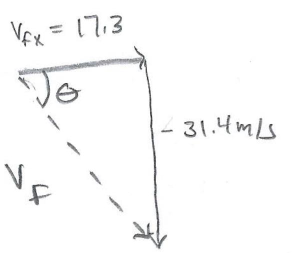
$V_{iy} = V_i \sin\theta = 20 \sin 30° = 10$ m/s

c) • $V_{fy} = V_{iy} + a_y t = 10 + (-9.8)(4.22\text{s})$

$\boxed{V_{fy} = -31.4 \text{ m/s}}$

• $V_{ix} = \boxed{V_{fx} = 17.3 \text{ m/s}}$ (b/c no accel in x-direction so velocity stays same)

we know the x- and y- component of final velocity, so we can find magnitude of final velocity.



$V_f = \sqrt{V_{fx}^2 + V_{fy}^2}$

$= \sqrt{(17.3)^2 + (+31.4)^2}$

$\boxed{V_f = 35.9 \text{ m/s}}$

$\theta = \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right) = \tan^{-1}\left(\frac{31.4}{17.3}\right)$

$\boxed{\theta = 61° \text{ below x-axis}}$